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ITC-2019: A MaxSAT approach to solve University Timetabling problems

Alexandre Lemos · Pedro T. Monteiro · Inês Lynce

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Abstract This paper describes an algorithm that uses a MaxSAT encoding to solve course timetabling and student sectioning problems as specified for the International Timetabling Competition 2019. We developed a hybridized algorithm that combines local search techniques and a MaxSAT solver.

Keywords ITC-2019 · MaxSAT · University Timetabling · Local Search

1 Introduction

The main goal of this submission is to propose a MaxSAT approach to solve university timetabling problems (Müller et al., 2018). In the past, Boolean Satisfiability (SAT) (Bittner et al., 2019), MaxSAT (Achá and Nieuwenhuis, 2014; Halaby, 2018) and local search (de Souza Rocha et al., 2012; Bellio et al., 2012; Müller and Murray, 2010; Lemos et al., 2019) approaches have been successfully applied for solving these problems.

Our approach uses the state-of-the-art solver *TT-Open-WBO-Inc* (Nadel, 2019a,b) that won both the Weighted and Incomplete categories at *MaxSAT Evaluation 2019*[1]. To improve performance, we apply a set of pre-processing methods to the input value. Moreover, to reduce the scope of the student sectioning portion of the problem, we also apply a neighborhood type search to further optimize the solution. Our implementation is available on github (https://github.com/ADDALemos/MPPTimetables).

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A. Lemos · P.T. Monteiro · I. Lynce
Instituto Superior Técnico, Universidade de Lisboa
INESC-ID, Rua Alves Redol, 9, 1000-029 Lisboa, Portugal
E-mail: {alexandre.lemos,pedro.tiago.monteiro,ines.lynce}@tecnico.ulisboa.pt

[1] The results are available at https://maxsat-evaluations.github.io/2019/index.html

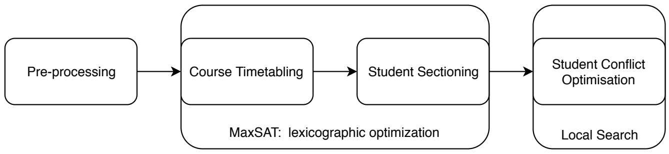

Fig. 1 Algorithm schema to solve the ITC-2019.

2 Algorithm

In this section, we describe our approach to solve the university timetabling problem described in (Müller et al., 2018). Figure 1, describes the overall schema of our algorithm.

2.1 Pre-processing

Our approach relies on two pre-processing methods to improve the performance of the overall method.

Identification of independent sub-problems of courses. In order to reduce the search space of the problem, we break an instance into self-contained sub-instances that can be solved independently without losing any solution.

Merge students with exactly the same course enrollment plan. In order to reduce the number of similar variables and constraints, we create groups of students sharing the same curricular plan. Furthermore, we limit the size of the group to the value of the greatest common divisor (Knuth, 1969; Schindl, 2019) between the total number of students enrolled in a course and the smallest capacity limit of the classes of that course. This process ensures that it is possible to find a feasible solution to the problem, since it is possible to combine all clusters of students into classes. But, it may remove the optimal solution by not allowing the assignment of an individual student to a given class.

2.2 MaxSAT approach

The university timetabling problem can be divided into two sub-problems: *course timetabling* (Di Gaspero et al., 2007) and *student sectioning* (Laporte and Desroches, 1986). We propose a MaxSAT encoding to solve the ITC-2019 problem, considering the linear search with the clusters algorithm (Joshi et al., 2018) implemented in *TT-Open-WBO-Inc*. The solver allows the constraints to be written as high level Pseudo-Boolean constraints which can be converted to SAT by different CNF encoding (Martins et al., 2014).

Moreover, the solver uses a lexicographical optimization criterion (Marques-Silva et al., 2011). The lexicographical order is computed based on the worst possible penalization of each criterion. An overview description of the encoding is presented below.

2.2.1 Course timetabling

The schedule of a class is described by three Boolean variables:

(v1) indicates which set of weeks from the class domain is assigned;
(v2) indicates which set of days from the class domain is assigned;
(v3) indicates which hour from the class domain is assigned.

The scheduling possibilities of a class are usually just a small part of the complete set of possible combination weeks, days and hours. Therefore, we only define these variables for acceptable values of the class domain. This way, we can reduce the number of Boolean variables always with the value 0. The assignment of a class to a room is represented by a Boolean variable.

The usage of these separated variables provides more flexibility when writing the constraints. This way, one can write the constraints with only the related variables (*e.g. SameDay* constraint will only affect the variables (v2) of the classes in question). Additionally, these separated variables also allow us to reduce the size of the constraints.

To solve course timetabling problems we need three different types of constraints:

(c1) Pairwise binary *at-least-one* constraints describing that two assignments cannot be part of the same solution;
(c2) *Exactly-one* constraints (*e.g.* a class must be scheduled in *exactly-one* hour) are encoded into CNF through the Ladder encoding (Ansótegui and Manyà, 2004);
(c3) Pseudo-Boolean constraints for the special constraints (*MaxDays* and *MaxDayLoad*). These constraints are later converted to SAT using the Adder encoding (Warners, 1998).

2.2.2 Student sectioning

The sectioning of a cluster of students into a class is represented by a Boolean variable. All hard constraints are encoded into SAT, ensuring that the solver finds a feasible solution to the problem. Note that the encoding requires auxiliary variables to deal with the possibility of multiple configurations in a course. Although, the scheduling conflicts may also be encoded into the MaxSAT problem as soft constraints, this is not practical. This part is performed by the local search phase of our approach.

2.3 Local search

The goal of this procedure is to improve the quality of the solution found without changing the schedule and room assignments of the courses. The local search procedure consists on a *swap* of students from class to class. Two types of moves are possible: swapping two students between classes, or moving a student to a class with empty seats. Considering these moves the local search procedure does not require the knowledge of hard constraints. Note, this procedure is only applied after a solution is found.

3 Summary and Future Work

Our MaxSAT approach allows us to find a solution to ITC-2019 problems in a short amount of time. The local search method allows us to locally improve the quality of the found solution.

As future work, we propose to cluster the classes into sub-problems with the same possible time assignments. This way, one could reduce the size of the search space and improve the quality of the found solution.

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